ST Statistics

Section 1: Calculus

Finite, countable and uncountable sets; Sequences of real numbers, convergence of sequences, bounded sequences, monotonic sequences, Cauchy criterion for convergence; Series of real numbers, convergence, tests of convergence, alternating series, absolute and conditional convergence; Power series and radius of convergence; Functions of a real variable: Limit, continuity, monotone functions, uniform continuity, differentiability, Rolle's theorem, mean value theorems, Taylor's theorem, L'Hospital rules, maxima and minima, Riemann integration and its properties, improper integrals;

Functions of several real variables: Limit, continuity, partial derivatives, directional derivatives, gradient, Taylor's theorem, total derivative, maxima and minima, saddle point, method of Lagrange multipliers, double and triple integrals and their applications.

Section 2: Matrix Theory:

Subspaces of $\mathbb{R}n$ and $\mathbb{C}n$, span, linear independence, basis and dimension, row space and column space of a matrix, rank and nullity, row reduced echelon form, trace and determinant, inverse of a matrix, systems of linear equations; Inner products in $\mathbb{R}n$ and $\mathbb{C}n$, Gram-Schmidt orthonormalization; Eigen values and eigen vectors, characteristic polynomial, Cayley-Hamilton theorem, symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal, unitary matrices and their eigen values, change of basis matrix, equivalence and similarity, diagonalizability, positive definite and positive semi-definite matrices and their properties, quadratic forms, singular value decomposition.

Section 3: Probability

Axiomatic definition of probability, properties of probability function, conditional probability, Bayes' theorem, independence of events; Random variables and their distributions, cumulative distribution function, probability mass function, probability density function and their properties, expectation, variance, properties of expectation and variance, moments and moment generating function, probability generating function, quantiles, distribution of functions of a random variable, Chebyshev, Markov and Jensen inequalities.

Section 4: Standard Univariate Distributions

Bernoulli, binomial, geometric, negative binomial, hypergeometric, discrete uniform, Poisson, continuous uniform, exponential, double exponential, gamma, beta distributions of first and second kinds, Weibull, normal, Cauchy.

Section 5: Joint Distributions

Jointly distributed random variables and their distribution functions, probability mass function, probability density function and their properties, marginal and conditional distributions, conditional expectation and moments, product moments, simple correlation coefficient, joint moment generating function, independence of random variables, functions of random vector and their distributions, distributions of order statistics, joint and marginal distributions of order statistics; multinomial distribution, bivariate normal distribution, sampling distributions: central chi-square, central t, and central F distributions.

Section 6: Convergence of Random Variables

Convergence in distribution, convergence in probability, convergence almost surely, convergence in $r-th$ mean and their inter-relations, Slutsky's lemma, Borel-Cantelli lemma; weak and strong laws of large numbers; central limit theorem for i.i.d. random variables.

Section 7:	Stochastic Processes

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson process, birth-and-death process, pure-birth process, pure-death process, Brownian motion and its basic properties.

Section 8:	Estimation

Sufficiency, minimal sufficiency, factorization theorem, completeness, completeness of exponential families, ancillary statistic, Basu's theorem and its applications, unbiased estimation, uniformly minimum variance unbiased estimation, Rao-Blackwell theorem, Lehmann-Scheffe theorem, Cramer-Rao inequality, consistent estimators, method of moments estimators, method of maximum likelihood estimators and their properties;

Interval estimation: pivotal quantities and confidence intervals based on them, coverage probability.

Section 9:	Testing of Hypotheses

Neyman-Pearson lemma, most powerful tests, monotone likelihood ratio (MLR) property, uniformly most powerful tests, uniformly most powerful tests for families having MLR property, uniformly most powerful unbiased tests, uniformly most powerful unbiased tests for exponential families, likelihood ratio tests, large sample tests.

Section 10:	Non-parametric Statistics

Empirical distribution function and its properties, goodness of fit tests, chi-square test, Kolmogorov-Smirnov test, run tests, sign test, Wilcoxon signed rank test, Mann-Whitney U-test, Kruskal-Wallis test, rank correlation coefficients of Spearman and Kendall.

Section 11:	Multivariate Analysis

Multivariate normal distribution: properties, conditional and marginal distributions, maximum likelihood estimation of mean vector and dispersion matrix, Hotelling's $T2$ test, Wishart distribution and its basic properties, multiple and partial correlation coefficients and their basic properties.

Section 12:	Regression Analysis

Simple and multiple linear regression, $R2$ and adjusted $R2$ and their applications, distributions of quadratic forms of random vectors: Fisher-Cochran theorem, Gauss-Markov theorem, tests for regression coefficients, confidence intervals.